Facts as figures

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A relation may be said to hold between one thing and another; thus one city is larger than another, a person is older than another, and so on. It's widely supposed that something may bear a relation not only to another thing, but also to itself. Someone may love herself or not; a villager may or may not shave that very villager. Now, if nothing could bear a relation to anything else, then there would be no need for the concept of relation in the first place: monadic properties would suffice. But presumably those monadic properties don't evaporate just because some things can bear relations to others. Some villagers still do shave and some don't, for example. To a given binary relation, then, there corresponds a monadic property which we may call its contraction.

A relation might also be supposed to hold of a couple of things in one order as opposed to another: one city may be larger than another and not vice versa. So it seems that a relation should be distinguished from its converse, that is, from the relation which would hold of the couple permuted. Russell (1984), Williamson (1985), Fine (2000), and Dorr (2004) have all contended that this is an illusion: that there really is no distinction between a relation and its converse. These arguments sparked some resistance, notably from MacBride (2007) and Liebesman (2013), (2014).

In this paper, I will show that the arguments of Russell *et al* are more radical than it looks. So relations would have less structure, not more, than even Russell *et al* have concluded. On the grounds which allegedly establish the indistinguishability of a relation and its converse, I will argue that a relation and its contraction cannot be distinguished either. This claim has a surprising consequence. If something can bear a relation to itself, then that relation must have a contraction. But since a relation and its contraction cannot be identical yet cannot be distinguished, it will follow that nothing bears a relation to itself.

Some earlier writers have considered the hypothesis that nothing bears a relation to itself, perhaps most prominently Armstrong (1978). But it's never found wide support. The hypothesis evokes misunderstandings familiar to logic teachers, like the presumption that two variables cannot assume the same value. A better reason is just that it should seem intuitively outrageous. Consider, for example, John in the supermarket, tracking a trail of sugar which spills from a shopper's cart. John knows himself to be chasing somebody. Eventually he's surprised. For though he's chased many things, his own self was never one of them. There's a first time for everything, isn't there? Surely, in this case, John the metaphysician could conclude that there's some relation R such that he knew himself to bear R to something; he could have turned out to be right not only because he was chasing another shopper, but also because he was chasing himself. I'll argue, in that case, that if the Russellians are right, then John the metaphysician would be mistaken. Yes, there may be predicates truly applicable both to John and another, as well as to John and himself. But, that will be precisely insofar as the predicate does not express a single relation at all.

1 The classical view of relations

The classical view of relations derives from an influential theory of truth for formal languages. In such a language, combining a predicate with a term yields a simple sentence. An interpretation finds the term to denote an object, and assigns the predicate some set of objects as extension. So interpreted, the sentence is true if and only if the extension contains that object. More generally, a sentence also results by combining a predicate with a sequence of terms. In that case, the sequence of terms determines a corresponding sequence of objects. The predicate is still assigned an extension; this time the extension isn't any old set but rather a set of sequences. Now the sentence is true iff the extension contains the sequence.

The success of this formal semantics motivates a corresponding conception of the structure of properties, relations, and their exemplification by objects. For it's antecedently natural to suppose a one-place predicate to express a property, so that the truth of an application of the predicate reflects that the property is exemplified by some object. And in an exactly parallel way, it's tempting to conclude that the truth of an application of a two-place predicate reflects the corresponding relation is borne by a sequence of objects. For example, the result of applying a predicate '$<$' to the sequence of terms '2', '3' is a truth, because the numbers two and three, in that order, do bear the relation of being less than. A sequence which results by reversing that order of numbers no longer exemplifies the less-than relation; and this is why in contrast the result of applying '$<$' to '3', '2' is a falsehood.

This extraction of metaphysics from semantics entails a substantial structural condition on the class of relations. Associated with the relation expressed by '$<$' is a so-called converse relation, expressed by '$>$'. For a couple of numbers to exemplify a relation in one order just is for the same pair in the other order to exemplify the converse. Since '2', '3' exemplify the less-than relation but '3', '2' don't, it follows that the less-than relation and its converse are distinct. The same holds of any predicate whose true application depends on the order of its arguments: it expresses a relation which has a distinct converse.

To bring out the meaning of this consequence, let's back up a bit and consider what's exactly at stake in understanding relations. It's widely supposed that a central question for philosophers is that of what exists, of what ultimately belongs in the furniture of the universe. But even granted an answer to that question of what there is, still there would remain a further question of how things are. Now even children generally appreciate that the one question "how is everything?" is too ambitious to tackle in its full generality. So instead people pursue particular questions to the effect of how is this or that. The philosophical term for this questionwise parcelled nonontological residue of the universe is "fact". So, it is to ask after the obtaining of a certain fact that a child asks whether the Pacific Ocean weighs more than the moon. It is to ask after the obtaining of another fact to wonder whether the moon weighs more than the sun.

It is clear that even about some fixed bunch of things, people can ask many questions. This indicates that the same things can constitute many facts. So beyond the mere constituents, whose existence may in general be acknowledged in the question already, the obtaining of a fact requires something more. But what? Russell (1903, 49) proposed that a question can't be expressed by a mere list of names, but must also include what he called a "verb". Still, it doesn't suffice merely to append the nominalization of a verb to the list of names—for it would remain unspecified what is being asked about that larger bunch of things. Rather, continues Russell, a verb yields a question when it "occurs as a verb". Perhaps this linguistic requirement reflects something in the nature of facts: just as constituents of a fact correspond to the names, so there corresponds, to the occurrence of a verb as a verb, some specified manner in which those constituents might hang together. This manner is a relation. The obtaining of the fact, over and above the mere existence of its constituents, would amount to their exemplifying a relation.

So, it would be the variety of relations which accounts for the variety of facts about a fixed bunch of things. To every such way for some things to hang together there would correspond a distinct fact. Contraposing, let's call this the principle of Uniqueness of Unifiers:

UU for any given fact, there's at most one relation, such that the obtaining of the fact amounts to that its constituents exemplify that relation.

The principle UU can be thought of as saying that inasmuch as questions about some bunch of things are substantively various, facts are at least that various too.

On the other hand, not every verbal variation is substantive. Surely some facts can be addressed both in French and in English. Surely the question whether the moon is heavier than the ocean is exactly decided by the fact that the ocean is lighter. These variations are merely verbal. What decides the truth of one could not arise without what decides the truth of the other, it couldn't cause anything the other doesn't, and so on. Yet, distinctions pertaining to what's "really out there" must find some modal, causal, or broadly explanatory realization. And this implies, conversely to UU, an upper bound on fact-individuation. As Russell put it:

> Looking away from everything psychological, and considering only the external fact in virtue of which it is true to say that A is before B, it seems plain that this fact consists of two events A and B in succession, and that whether we choose to describe it by saying "A is before B" or by saying "B is before A" is a mere matter of language. Owing to the fact that speech is in time and writing in space, we must mention A before mentioning B…(Russell and Eames, 1984, 85).

So, Russell compares facts

$$A \text{ precedes } B$$
$$B \text{ succeeds } A$$

and says they're really just one. So by UU, the relation of preceding must be identical with the relation of succeeding. The moral is fairly general: if distinctness of unifiers implies distinctness of facts, then realism about facts implies that there is no genuine difference between a relation and its converse.

If this simple conclusion is right, then the question arises why it might not have seemed so, and, for that matter, why it's been widely resisted. In the passage just quoted, Russell speculates that the apparent difference between a relation and its converse originates in a contingent feature of much linguistic representation, its linearity. Kit Fine illustrates Russell's hypothesis with a miniature language:

> the amatory predicate of the language might be a heart-shaped body that is red on one side and black on the other. To say that one person loves another, we then inscribe the name of the lover on the red side and of the beloved on the black side. 2000, 6.

Since there is no conventional ordering of these occurrences of names, the paper sentence does not represent lover and beloved as ordered either. Yet, it's nonetheless plausible that nothing is relevantly lost in translation from English to cardiac. The representation of objects as in an order disappeared with the order in their names. The disappearance suggests that the impression of order in the represented facts "out there" originates in a pervasive but contingent feature of linguistic representation.

2 Positions

We've now seen that the classical view, whereby relations apply to objects in some specific order, conflicts with intuitions which inspire the metaphysics of relations in the first place. One might simply reject those intuitions, and deny the assumption UU that each fact has at most one unifier. Following Russell and Eames (1984) and Fine (2000), however, it's my aim here to consider the consequences of respecting them. It might also be denied that any asymmetric relations exist (Dorr, 2004). But before taking such drastic measures, it seems reasonable to explore alternative accounts of exemplification which do not invoke an ordering of the relata. In that case, the problem remains how to distinguish the two ways in which two objects can exemplify an asymmetric relation.

The classical view is that two objects don't exemplify a relation neat, but only with respect to some ordering. But an ordering is just a labelling of objects by numbers. This suggests a surgical correction to the classical view: allow labels which aren't intrinsically ordered. Let's refer to arbitrary, possibly unordered multiplicities of labels as positions. Thus, a relation carries with it a set of positions, and it's as objects assume those positions that the relation is exemplified. In particular, the relation of loving carries the positions of lover and beloved. And that relation is exemplified, for example, as Marcel is assigned the position of lover and Albertine that of beloved—but not vice versa. Since there's no logical reason to suppose that the position of lover precedes that of beloved or vice versa, the positionalist account avoids ascription of order.

As Fine points out, this proposal appears to give an incorrect account of strictly symmetric relations, like for example adjacency.[1] A strictly symmetric relation would, on the positionalist account, be exemplified by two objects relative to an assignment of one object to one position and the other to the other. But then there is also another way in which those objects can exemplify the relation, namely by exchanging the positions they occupy. Yet this surely corresponds no variety in exemplification itself. So the positionalist account falls afoul of a milder form of the problem of "too much structure" which undermined the classical view in the first place.

3 Substitution

At this point, there still remains room for further theories of internal constitution of facts to explain the diversity of exemplification. But let's first take a step back and reflect on the explanandum structure which frustrated the classical and positionalist analyses. Fundamentally, it consists of a class of objects, and a class of facts. On top of these two classes, there seems to be further structure like this: that as some objects o, p figure in one fact, so objects q, r may figure in another. It's tempting to suppose that such a commonality, which we might call cofiguring, is what underlies the reidentification across facts of this or that relation.

Thus, the defects of the classical and positionalist views can be understood as erroneous implications about the individuation of manners in which objects figure in facts. Why not simply take the notion of cofiguringn as primitive? Fine remarks that cofiguring "is not the sort of notion which can be taken as primitive" (2000, 25), perhaps because of its baroque logical type.On the other hand, Fine continues:

> ...we have a general understanding of substitution, one that is not tied to any particular domain of application. Thus, we can understand what it is to substitue one expression for another, or one element of a set for another, and so on. (Fine, 2000, 26)

So the concept of substitution might form the basis of an account of relations which is adequate to their structure.

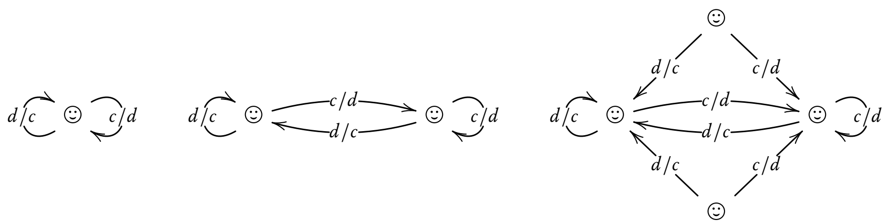
To see how this might proceed, let's consider a variety of substitution which is simple and concrete. Suppose that A is a string, or finite sequence of characters. Then, substituting character d for character c in A amounts to replacing each occurrence of c with an occurrence in that same position the character d.

The concept of substitution by itself suffices to express various other notions about strings. For example, for a character o to occur in a string A just is for it to be possible to change A by replacing o with something else. More importantly, the concept of substitution explains the number of times a character occurs in a string. [2] In turn, the account of recurrence yields an account of a string's length: the length of a string is just the sum of the numbers of occurrences of its constituents.

[1] A relation R is strictly symmetric if for it to be the case that Rab just is for it to be the case that Rba.

[2] Suppose, for example, that A contains at least two occurrences of o. This can be taken to consist in the existence of distinct objects p and q, and distinct strings B and C, such that replacing q with o changes C into B, and replacing p with o changes B into A.

To bring out just what substitution does articulate, note that one can visualize the structure of substitution on complexes as a graph. Nodes represent complexes, and arrows colored by pairs of objects represent accessibility between complexes by substitution. As far as substitution can tell, the string cd and its "converse" dc stand in distinct positions in the web of substitution-accessibility, but the positions are perfectly symmetric. More generally, say that an anagram of a string A is a result of permuting the positions of the occurrences of characters. It is clear that substitution relations are invariant under anagrams. As applied in the realm of strings, the concept of substitution represents anagrams as distinct but indiscernible. Here is a picture.



4 Contractions

We've now seen three accounts of relations. The classical approach distinguished a relation from its converse. The position-based account still distinguished two ways of exemplifying a symmetric relation. After stepping back to survey what we're trying to explain, in particular the concept of cofiguring, we then saw Fine propose to take as primitive the notion of substituting an object for another in a fact. It is, of course, possible to give an account of cofiguring in the classical or positionalist frameworks; the difficulties arose in that they posit not too much but rather too little structure. Now I now want to consider whether the framework of substitution is in fact adequate to the structure of the explanandum.

Let's begin with a couple of examples. A number is said to be perfect if it is the sum of its own proper factors. Thus, for example, a fact

the number six is perfect

does obtain, because six is such that its proper factors total six. In this respect, six is unusual. The sum of the proper factors of four is three; the proper factors of five total one. Six, in other words, bears to itself a relation which most numbers bear to another.

Similarly consider, for example, a circumstance to the effect that

Alfred is shaving.

This seems to consist of one person doing a single activity, and thus of something enjoying a monadic property. But then Alfred might go to work at the barber shop, and eventually bring it about that

Alfred is shaving Louis.

It seems as though Alfred's shaving should now be redescribed as a case in which Alfred was shaving himself, rather than somebody else.

Intuitively, the idea behind these examples is this. To a binary relation R there may correspond a unary relation R' such that for something to bear R to itself just is for it to exemplify R'. Let's call R' a contraction of R. And let's say that a completion of a relation R is a fact F such that for some objects oo, the obtaining of F consists in the exemplification of R by oo. About contractions the following then holds, as a mere matter of words: that a completion of a contraction of a relation is a completion of the relation.

Recall, from §1, that the principle UU says that the constituents of any fact are related, in the fact, by at most one relation. Suppose, now, that the contraction R' of R has a completion, say C. Then C is also a completion of R. So by UU, R and R' are identical. Thus, UU implies that a relation is identical to each of its completable contractions.

In §1, we saw that for a broad range of cases, the principle UU undermines the distinction between a relation and its converse. Since the presumption of ordering of relata underwrites such a distinction, it follows from UU that relata need not be ordered after all. Here, I've argued that the principle UU undermines the distinction between a relation and its completable proper contractions.

Now, the notions of converse and of contraction are disanalogous in an important respect, which indicates that one response to the problem of converses does not carry over to the present situation. From the conclusion that there is no difference between a relation and its converse, Williamson (1985) inferred the apparent pair of relations must really be a single one. Now it may not be so hard to identify a relation with its unary contraction, for the unary contraction is unique. But a relation of arity greater than two would seem to have several binary proper contractions. By the transitivity of identity, all these binary relations would be the same too. There would be just a single binary relation borne by Isabella to an art collection in the facts

Isabella gave herself to the collection
Isabella gave the collection to herself.

So it would seem that a relation and its proper contractions cannot in general be identified. In the present case, we can't just identify the indistinguishable.

On the other hand, Fine (2000) advocated another diagnosis: that the concept of converse just does not apply to relations satisfying UU. Presumably, the grounds of such a diagnosis would be that the concept of converse depends on the notion of ordering of the relata, which was then rejected. So, let's now consider the basis of the concept of contraction.

5 Arity

The concept of contraction was explained by appeal to a concept of arity. Thus, a proper contraction of a binary relation R is a unary relation R' such that for something to exemplify R' just is for that thing to bear R to itself. What, however, are these underlying notions expressed by "unary", "binary", and so on? Let's turn to the question how these notions might be explained in the three frameworks of §§1-3.

In the classical framework, each relation holds or not of some objects in some specified order. An order is an assignment of objects to some initial segment of the ordinals. Then the concept of arity can be explained as the length of that segment.[3] In the positionalist generalization the case is similar. Each relation would carry a set of positions, and the arity of a relation would be the cardinality of that set.

Let's now consider standing of the concept of arity in the substitutional framework. This isn't quite so straightforward, because the account of Fine (2000) doesn't spell out, in terms of substitution, when two facts do have a relation in common. However, Fine notes that without transfactual object-positions, it no longer makes sense to talk about substituting one relation for another in a fact (2000, 30).[4] So a relation might be thought of as some core which is inalterable by substitution. More precisely, we might say that two facts A and B have the same unifying relation if they are connected by substitutions, where substitutions may change direction, as in …

$$A \leftarrow\!\shortmid C \mapsto B.$$

So understood, substitutionalism does interpret the concept of arity. For example, say that a substitution σ is invertible on A if there's a substitution τ such that $A\sigma\tau = A$. Then, the number of recurrences of objects in A is the length of the longest chain of noninvertible substitutions which eventuates in A; and the number of occurrences is the sum of the constituents and the recurrences. Finally, the arity of a relation in A can be taken to be the number of occurrence of objects in A.

So, a substitutional framework which countenances noninvertible substitutions would interpret the concept of arity. Thus, the puzzle of §4 can be stated substitutionally. How does it fare? The puzzle starts with the hypothesis that a binary relation R has a proper contraction R', where R' has smaller arity than R, and where facts united by R' just are facts united by R. Now, the substitutional interpretation of arity implies that no proper contraction can be exemplified. So the puzzle does not get off the ground.

But this, however, is out of the frying pan and into the fire. If no proper contraction can be exemplified, then every apparent exemplification of a proper contraction requires a choice. A fact that six totals its proper factors, or that Alfred shaves, must have one of the forms Fa and Raa. As I'll now argue, such a choice is unintelligible.

In the literature about converses, Dorr (2004) considers a possible world in which two ontologically basic relations confer isomorphic linear orderings on the same bunch of simple particulars. Are those orderings the same, or is one the reversal of the other? Dorr claims that each of these options is logically possible, but that there are no grounds on which distinguish them.

Now consider instead the actual world, or some close relative of it, in which Alfred is shaving. Is this a place with a unary relation Alfred thereby exemplifies, or a place

[3] The classical framework might be liberalized, so that a relation is a set of sequences of various lengths. In that case, arity will be a many-valued function. But this still allows the concept of contraction to be explained as a transformation on sets of sequences.

[4] For example, what is the result of substituting the relation of being older than (which is the same as the relation of being no younger than) for whatever relation occurs in the fact that a is heavier than b?

with a binary relation which Alfred bears to himself? It's not clear it could be that each of these two exclusive options is logically or metaphysically possible. Still, they are surely in some sense both epistemically possible: for, we have no idea which is the one that actually obtains. And that is no accident, for the supposition that it is one rather than the other is unintelligible.

Thus, the substitutional framework raises questions that ought not exist, like: does Alfred's shaving result from substituting him for Louis in the fact that Louis is shaving him? Fine did suggest "we have a general understanding of substitution"; perhaps the understanding is quite so general after all.

The problematic notion of arity can be factored into two other notions: constituent and recurrence. It seems hard to imagine anything like a Russellian conception of fact without a notion of constituent. And indeed, the source of problems would seem to be intrafactual recurrence.

This analysis of the problem suggests the idea of restricting to substitutions which are invertible. In that case, the concept of recurrence becomes trivial. The resulting notion of arity would derive from the bare objectual multiplicity of constituents of a relation's completion. And this would dispell the unintelligible choices.

Kripke has had some choice words for the view that "a relation, being essentially two-termed, cannot hold between a thing and itself":

> somebody can be his own worst enemy, his own severest critic, and the like. [...] Some relations are reflexive such as the relation 'no richer than.' Kripke (1980, 108n)

If the linguistic evidence were so clear as Kripke suggests, then it should be obvious that the sentences

Everybody dislikes Donald.
But Donald only dislikes people who are louder than he is.

entail

Donald is louder than himself.

Likewise, "who shaves Martin?" might well be answered "nobody: he shaves himself." Here the contradiction in the second sentence—if there were one at all—would seem to be patent. In both cases, one account of the obscurity is that people detect some tacit equivocation in the predicate. Thus, Kripke's linguistic evidence seems at best equivocal.

Kripke's second argument is that in general, a reflexive relation can be found from an irreflexive one, as with the relation of being no richer than. This method works by manufacturing a reflexive relation from an irreflexive one; and so it shows that the proffered reflexivity is not primitive. Such non-primitive relations could be explained away as mere extensions of logically complex predicates.[5]

[5] In this way, for example, no formula in the language of set theory, or even all of mathematics, can yield an incontrovertible example of irreducibly variadic relation, because the only relations required for the interpretation of the defining formulas will be membership and identity, but neither of these is objectually variadic.

Nonetheless fixing the number of relata yields a standard of individuation of relations which may seem too stringent. In particular, it's tempting to suppose that there's a single relation in virtue of which any multiplicity of people would be lined up. But, for example, it's widely believed that the size of such multiplicities does vary between checkout counters at the supermarket. Such facts with correspondingly variable numbers of constituents would not be linked by invertible substitution. This is not just a shortcoming of the purely invertible form of the substitutional framework for relational individuation. Applying a noninvertible substitution to a lineup of people would land someone in the confusing predicament of being in front of or behind her own self.

Let me summarize the upshot of the considerations so far. The classical and positionalist accounts of relations fail because they posit too much structure. The classical account distinguishes a relation and its converse; the positionalist account still distinguishes two exemplifications of strictly symmetric relations. An elegant and plausible response is to try individuating relations by reference to some sort of transformation on their completions. However, the general concept of substitution still yields an interpretation of the concept of arity, thereby requiring choices which do not seem genuine. That is, so individuating relations by sometimes noninvertible substitution still posits too much structure. When the class of substitutions is restricted to those which are invertible, the notion of arity collapses into that of bare objectual multiplicity.

6 Recurrence

The notion of arity is the product of the notions of constituent and of intrafactual recurrence. As I've urged, the notion of constituent is essential to a Russellian conception of fact. So it's about the notion of recurrence that we should look askance. How does it originate? Russell suggested that the appearance of an order of relata originates in a pervasive but contingent feature of many representational systems, namely linearity. I'll conclude by suggesting that the concept of recurrence of an object in a fact originates in a feature of representation which is even more pervasive, the type-token distinction. Still it can be exposed as such, for the feature is not quite essential.

Suppose, first, that Alice wants to tell the court how some wooden blocks were arranged on a table at the scene of a crime. She might, of course, produce a string of words:

"c was in front of d and d was in front of e." (1)

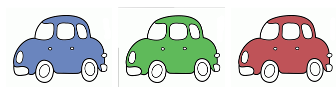
Insodoing the term "d" occurs twice—both in the string itself, and also in the underlying parse tree.

But upon cross-examination, Alice might be given those blocks and that table. And she'd make an arrangement:

(2)

In Kit Fine's language of hearts, there was no reason to suppose that the red side of a paper heart is in some sense 'first', while the black side is 'second'. It would be equally arbitrary to suppose that green block occurs twice on the table while the blue block occurs only once. Yet surely, nothing is lost in translation from (1) to (2). The witness's testimony upon cross-examination is if anything clearer; Alice brings the jury actually to see what she's thereby telling them.

Now, it might be objected that a replica is not automatically a representation, and maybe the block-arrangement in the courtroom, in merely replicating the arrangement at the crime-scene, therefore doesn't really represent.[6] However, we might instead suppose the crime to have occurred in traffic. Then (2) might be used to represent an arrangement


. (3)

Of course, this confers no intelligibility to the hypothesis that any blocks recur. Nor is there any feeling that some representation of recurrence in the cars has now been lost; after all a traffic "accident" could be contrived to represent a witness's testimony instead.

More generally, granted only one token representative of each individual thing that there is, it would still be possible in principle to represent any particular fact. At some point, people would presumably start fighting over whose turn it was to mention this or that, and somebody would start printing duplicates. But then again, even in disagreeing over who gets to mention the car, two people would mention the car already, and perhaps even planning one's turn would entail some covert duplication of proxies.

In other words, recurrence is an almost irrepressible but still contingent feature of representational structures. Predicates, in particular, are a sort of thing which it makes sense to talk about as having arity. For something is a predicate by belonging to some system of rules of constructing sentences. And a sentence is something like a tree plus an assignment of positions to vocabulary.[7] But the recurrence of proxies in a representation can be eliminated without any compensating introduction of anything like numerals, yet also without loss of content. This indicates that the content is not characterized by recurrence. That is, recognition of the structure of a fact does not require acknowledging internal recurrence of constituents. If that's right, then we can have no basis for supposing objects recur in a fact at all.

[6]For example, surely there's no sense in which counterfeits represent Franklins. But, it seems that counterfeits do not represent because there's some kind of purpose which they're not assigned; and that's seemingly not what's lacking in the courtroom blocks. Perhaps there is in some sense not enough "distance" between the two facts, because the blocks don't stand for something else. It doesn't follow, though, that they don't stand for anyhing, any more than the words in this sentence can't be used to stand for this very sentence.

[7]Note that this is not to conclude that there are some facts, namely sentences, about whose structure positionalism does give the correct theory. For example, it might be maintained that a sentence is a complex, whose constituents include both parse tree and its positions, as well as words. Alternatively, it may be that unlike complexes in general, sentences have structural features which allow a relative interpretation of the theory of positions; then positionalism would be "as good as true" for sentences if not true *au pied de la lettre*.

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